$\frac{n(n+1)}{2}$ $\frac{n(n+1)(2n+1)}{6}$

Summation Properties Worksheet

Directions. Use the *properties and formulas* discussed in class to evaluate each of the following summations, and answer the related questions.

1. $\sum_{i=1}^{14}(3i+2)$

$3\sum_{i=1}^{14} i + \sum_{i=1}^{14} 2$

$3\frac{(14)(15)}{2} + 2(14) = \boxed{343}$

2. $\sum_{i=1}^{12}(-2i^2+6)$

$-2\sum_{i=1}^{12} i + \sum_{i=1}^{12} 6$

$-2\left(\frac{12(13)(25)}{6}\right) + 12(6) = \boxed{-1228}$

3. $\sum_{k=1}^{22}(-2k+5)$

$-2\sum_{k=1}^{22} k + \sum_{k=1}^{22} 5$

$-2 \cdot \frac{22(23)}{2} + 22(5) = \boxed{-396}$

4. $\sum_{i=1}^{10}(i^3-2)$

$\sum_{i=1}^{10} i^3 - \sum_{i=1}^{10} 2$

$\frac{10^2(11)^2}{4} - 2(10) = \boxed{3005}$

5. $\sum_{n=8}^{24}(7n-8)$

$7\sum_{n=1}^{24} n - 24(8) - \left(7\sum_{n=1}^{7} n - 7(8)\right)$

$\left[7\frac{(24)(25)}{2} - 24(8)\right] - \left[7\frac{(7)(8)}{2} - 7(8)\right]$

$(2100 - 192) - (196 - 56)$

$1908 - 140$

$\boxed{1768}$

6. $\sum_{i=11}^{38} 18$

$28(18) = \boxed{504}$

7. $\sum_{i=1}^{18}(i-3)^2$

$i^2 - 6i + 9$

$\sum_{i=1}^{18} i^2 - 6\sum_{i=1}^{18} i + 18(9)$

$\frac{18(19)(37)}{6} - 6\frac{(18)(19)}{2} + 162$

$2109 - 1026 + 162 = \boxed{1245}$

8. $\sum_{i=1}^{12}[(i-7)(i+7)]$

$i^2 - 49$

$\sum_{i=1}^{12} i^2 - \sum_{i=1}^{12} 49$

$\frac{12(13)(25)}{6} - 12(49)$

$650 - 588 = \boxed{62}$

$1 \quad 3 \quad 3 \quad 1$

$784 - 840 + 336 - 56 = \boxed{224}$

$$\left[.7\sum_{i=1}^{147} \frac{(147)(148)}{2} - 32(147)\right] - \left[.7\sum_{i=1}^{88} \frac{88(89)}{2} - 32(88)\right]$$

9. $\sum_{a=1}^{7}(a-2)^3$

$a^3 - 6a^2 + 12a - 8$

$$\sum_{a=1}^{7} a^3 - 6\sum_{a=1}^{7} a^2 + 12\sum_{a=1}^{7} a - 7(8)$$

$$\frac{7^2(8)^2}{4} - \frac{6(7)(8)(15)}{6} + \frac{12(7)(8)}{2} - 56 \quad \boxed{224}$$

784

10. $\sum_{i=89}^{147} 0.7i - 32$

$(7614.6 - 4704) - (2741.2 - 2816)$

$2910.6 - (-74.8)$

$\boxed{2985.4}$

11. Consider the series $-6-2+2+6+\cdots$. Find the number of terms, n, if $S_n =$ 640.

$4 \quad 4 \quad 4$

$a_n = -6 + 4(n-1)$

$a_n = -6 + 4n - 4$

$a_n = -10 + 4n$

$640 = \frac{n}{2}(-6 + (-10+4n))$

$640 = \frac{n}{2}(-16+4n)$

$0 = 2n^2 - 8n - 640$

$0 = 2(n^2 - 4n - 320)$

$0 = 2(n-20)(n+16)$

$\boxed{n=20}$ $n = \cancel{-16}$

12. Consider the series $-6-3+2+9+\cdots$. Find the number of terms, n, if $S_n =$ 1384.

$3 \quad 5 \quad 7$

$2 \quad 2$

$$\sum_{i=1}^{n} i^2 - 7$$

$$6\left(\frac{n(n+1)(2n+1)}{6} - 7n = 1384\right)$$

$(n^2+n)(2n+1) - 42n = 8304$

$2n^3 + 2n^2 + n^2 + n - 42n = 8304$

$2n^3 + 3n^2 - 41n - 8304 = 0$

doesn't factor; graph to find zeros

$\boxed{n=16}$

see next page

13. Consider the series $212+199+186+173+\cdots$. Find the number of terms, n, if $S_n = -$4488.

$13 \quad 13 \quad 13$

14. Simplify

$$\sum_{k=0}^{n} \left(k^2 - (k+1)^2\right)$$

15. Simplify

$$\sum_{k=1}^{n} \left(\frac{1}{k+1} - \frac{1}{k}\right)$$

14. $\sum_{k=0}^{n} \left(k^2-(k+1)^2\right)$

$\sum_{k=0}^{n} k^2 - (k^2+2k+1)$

$\sum_{k=0}^{n} \cancel{k^2}-\cancel{k^2}-2k-1$

$\sum_{k=0}^{n} -2k-1$

If $k=0 \rightarrow -1$, so

$-1+\sum_{k=1}^{n} -2k-1$

$-1 + -\cancel{2}\frac{(n)(n+1)}{\cancel{2}} - n$

$-1-n^2-n-n$

$-n^2-2n-1$

$\boxed{-1(n^2+2n+1)}$

15. $\sum_{k=1}^{n} \frac{1}{k+1} - \frac{1}{k}$

$\sum_{k=1}^{n} \frac{1}{k+1} - \sum \frac{1}{k}$

$\frac{1}{\frac{n(n+1)(2n+1)}{6}} - \frac{1}{\frac{n(n+1)}{2}}$

$\frac{6}{n(n+1)(2n+1)} - \frac{2}{n(n+1)} \cdot \frac{2n+1}{2n+1}$

$\frac{6}{n(n+1)(2n+1)} + \frac{-4n-2}{n(n+1)(2n+1)}$

$\frac{-4n+4}{n(n+1)(2n+1)}$

$\boxed{\frac{-4(n-1)}{n(n+1)(2n+1)}}$

13

$-4488 = \frac{n}{2}\left(212+\underline{225-13n}\right)$

$2\left(-4488 = \frac{n}{2}(437+13n)\right)$

$-8976 = {}^{-}13n^2+437n$

$0 = {}^{-}13n^2+437n+8976$

doesn't factor - quad form or graph in calc to find roots

$a_n = 212+(n-1)^{-}13$

$a_n = 212-13n+13$

$a_n = 225-13n$

$\boxed{n=48}$