

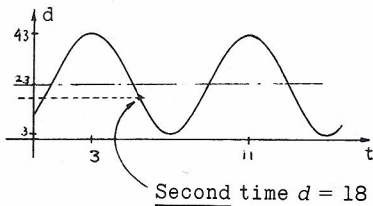
## Exercise 2-11, 2-12

|     |            |
|-----|------------|
| 3   | 3.5        |
| 3.5 | 3.92836283 |
| 4   | 4.29813333 |
| 4.5 | 4.59807621 |
| 5   | 4.81907786 |
| 5.5 | 4.95442326 |
| 6   | 5          |
| 6.5 | 4.95442326 |
| 7   | 4.81907786 |
| 7.5 | 4.59807621 |
| 8   | 4.29813333 |
| 8.5 | 3.92836283 |
| 9   | 3.5        |
| 9.5 | 3.02606043 |
| 10  | 2.52094453 |

EXERCISE 2-12, page 59; Sinusoidal Functions as Mathematical Models

### 1. Ferris Wheel Problem

(a) Graph.



(b) The lowest you go is 3 feet above the ground, because seats in a Ferris wheel do not scrape the ground.

(c) Let  $d$  = no. of feet above ground.

$$d = C + A \cos B(t - D)$$

$$C = 23, A = 43 - 23 = 20,$$

$$B = 2\pi/(11 - 3) = \pi/4, D = 3.$$

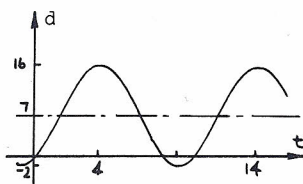
$$\therefore d = 23 + 20 \cos \frac{\pi}{4}(t - 3)$$

- (d) (i)  $d = 23 + 20 \cos \frac{\pi}{4}(6 - 3) \approx 8.86$  ft  
 (ii)  $d = 23 + 20 \cos \frac{\pi}{4}(4\frac{1}{3} - 3) = 33$  ft  
 (iii)  $d = 23 + 20 \cos \frac{\pi}{4}(9 - 3) = 23$  ft  
 (iv)  $d = 23 + 20 \cos \frac{\pi}{4}(0 - 3) \approx 8.86$  ft

(e)  $23 + 20 \cos \frac{\pi}{4}(t - 3) = 18$   
 $t = 3 + \frac{4}{\pi} \cos^{-1}(-0.25)$   
 $t = 5.321... + 8n$  or  $0.678... + 8n$   
 Positive values of  $t$  are about 0.68, 5.32, 8.68,  
 $\therefore$  Second time  $d = 18$  is  $t \approx 5.32$  sec.

### 2. Steamboat Problem

(a) Graph



(b) Let  $d$  = no. of ft above water.  
 Let  $t$  = no. of seconds.

$$d = C + A \cos B(x - D)$$

$$C = 7, A = 9, B = 2\pi/10 = \pi/5$$

$$D = 4$$

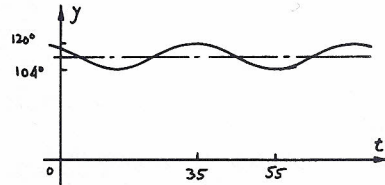
$$\therefore d = 7 + 9 \cos \frac{\pi}{5}(t - 4)$$

- (c) (i)  $d = 7 + 9 \cos \frac{\pi}{5}(5 - 4) \approx 14.28$  ft  
 (ii)  $d = 7 + 9 \cos \frac{\pi}{5}(17 - 4) \approx 4.22$  ft

- (d)  $7 + 9 \cos \frac{\pi}{5}(t - 4) = 0$   
 $t = 4 + \frac{5}{\pi} \cos^{-1}(-7/9) = 7.918... + 10n$  or  
 $0.081... + 10n$   
 First positive time is 0.08 sec, coming out (see graph).

### 3. Extraterrestrial Being Problem

(a) Graph.



(b) Let  $y$  = no. of degrees temp.  
 Let  $t$  = no. of min.

$$y = C + A \cos B(t - D)$$

$$C = \frac{1}{2}(120 + 104) = 112,$$

$$A = 120 - 112 = 8,$$

$$B = 2\pi/40 = \pi/20, D = 35$$

$$\therefore y = 112 + 8 \cos \frac{\pi}{20}(t - 35)$$

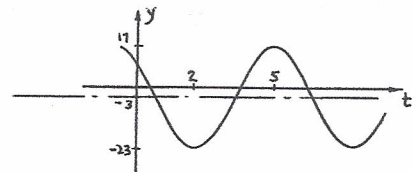
(c)  $y = 112 + 8 \cos \frac{\pi}{20}(0 - 35) \approx 117.7^\circ$

(d)  $112 + 8 \cos \frac{\pi}{20}(t - 35) = 114$   
 $t = 35 + \frac{20}{\pi} \cos^{-1} 0.25 = 43.39... + 40n$  or  
 $26.60... + 40n$   
 First positive times are 3.39 min, 26.61 min, 43.39 min.

3

### 3. Tarzan Problem

(a) Graph.



(b)  $y = C + A \cos B(t - D)$   
 $C = \frac{1}{2}(17 + (-23)) = -3,$   
 $A = 17 - (-3) = 20, D = 5,$   
 Per. =  $2(5 - 2) = 6, B = 2\pi/6 = \pi/3.$   
 $\therefore y = -3 + 20 \cos \frac{\pi}{3}(t - 5)$

- (c) (i)  $y = -3 + 20 \cos \frac{\pi}{3}(2.8 - 5) \approx -16.38$  m  
 (ii)  $y = -3 + 20 \cos \frac{\pi}{3}(6.3 - 5) \approx 1.16$  m  
 (iii)  $y = -3 + 20 \cos \frac{\pi}{3}(15 - 5) = -13$  m

(d)  $y = -3 + 20 \cos \frac{\pi}{3}(0 - 5) = 7$  m

(e)  $-3 + 20 \cos \frac{\pi}{3}(t - 5) = 0$   
 $t = 5 + \frac{3}{\pi} \cos^{-1} 0.15 = 6.356... + 6n$  or  
 $3.643... + 6n$

Least positive value is  $6.356... - 6$ , or about 0.36 sec.