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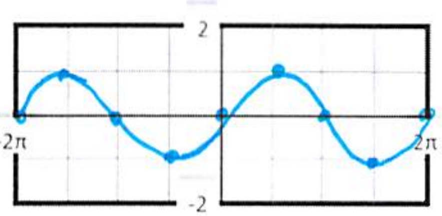
(FIRST 3 PAGES ONLY)

Graphing Inverse Trigonometric FUNCTIONS Name Doc

Part I. Sketch the graph of the sine, cosine, and tangent functions from [-2π, 2π]. Label the 9 consecutive critical points (x, y) for over that domain. Then, provide the domain and range for each function.

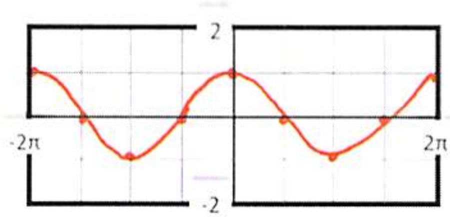
1. $y = \sin(x)$



X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	0	1	0	-1	0	1	0	-1	0

D: $(-\infty, \infty)$ R: $[-1, 1]$

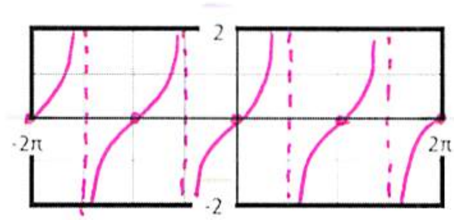
2. $y = \cos(x)$



X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	1	0	-1	0	1	0	-1	0	1

D: $(-\infty, \infty)$ R: $[-1, 1]$

3. $y = \tan(x)$



X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	0	A	0	A	0	A	0	A	0

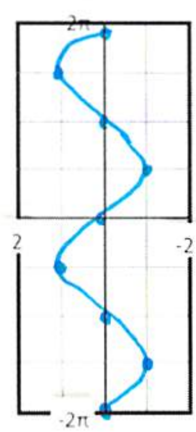
D: All real numbers except $\frac{\pi}{2}$'s R: $(-\infty, \infty)$

Part II. Inverse Relations. Functions and their inverses have a couple of important properties that you should remember. **(1)** The graphs of inverse functions are reflections through the line $y = x$. This means **(2)** that the x-values and the y-values from the function switch for the inverse. This implies that any point (a, b) in the function becomes the point (b, a) for the inverse. This further implies that anything horizontal becomes vertical and anything vertical becomes horizontal. Thus, the x-axis and y-axis switch (including their scale), as well as the domain and range.

Using these properties, (A) make a table of the critical values for the inverse relations, (B) sketch a graph of the inverses of each function in #1-3 above.

4. $y = \sin^{-1}(x)$

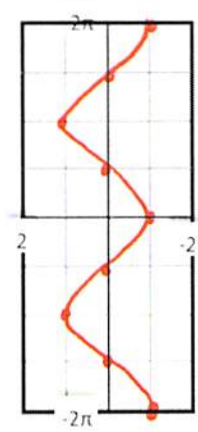
X	Y
0	-2π
1	$-\frac{3\pi}{2}$
0	$-\pi$
-1	$-\frac{\pi}{2}$
0	0
1	$\frac{\pi}{2}$
0	π
-1	$\frac{3\pi}{2}$
0	2π



D: $[-1, 1]$ R: $(-\infty, \infty)$

5. $y = \cos^{-1}(x)$

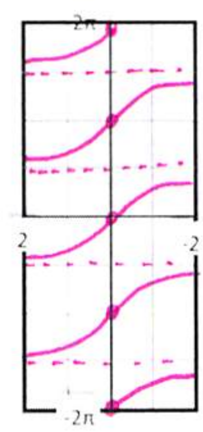
X	Y
1	-2π
0	$-\frac{3\pi}{2}$
-1	$-\pi$
0	$-\frac{\pi}{2}$
1	0
0	$\frac{\pi}{2}$
-1	π
0	$\frac{3\pi}{2}$
1	2π



D: $[-1, 1]$ R: $(-\infty, \infty)$

6. $y = \tan^{-1}(x)$

X	Y
0	-2π
A	$-\frac{3\pi}{2}$
0	$-\pi$
A	$-\frac{\pi}{2}$
0	0
A	$\frac{\pi}{2}$
0	π
A	$\frac{3\pi}{2}$
0	2π



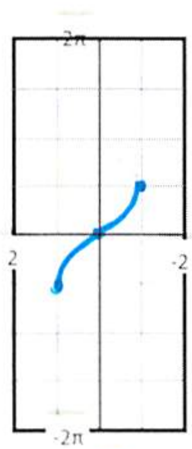
D: $(-\infty, \infty)$ R: all real #'s except $\pi/2$'s

Part III. ARCFUNCTIONS. The inverse of a trigonometric function is defined as the **arcfunction** of a trigonometric function when we limit the domain/range such that the inverse is also a function. This function must cover all possible inputs (which were the outputs/range of the original trigonometric function).

Examine the graphs of #4-6 above. Determine a section of the graph that covers all of the possible inputs covered by the graph (Domain). **HINTS:** Be sure to include the input $x = 0$ in the section you choose. Do your best to include the most possible positive inputs possible. And, do your best to make sure that the section of the graph you choose is continuous (can be drawn without having to lift your pencil).

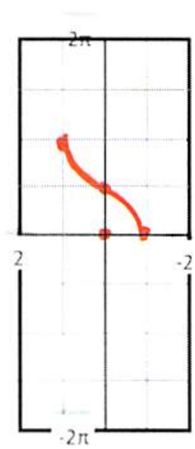
Sketch the graph of the section you chose (using the guidelines above) here.

7. $y = \arcsin(x)$



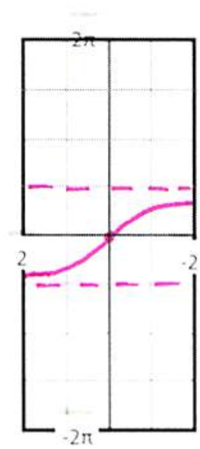
D: $[-1, 1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

8. $y = \arccos(x)$



D: $[-1, 1]$ R: $[0, \pi]$

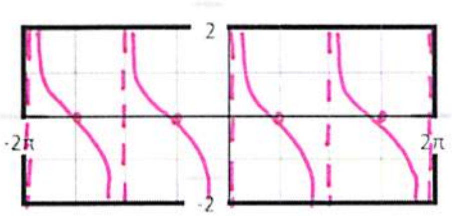
9. $y = \arctan(x)$



D: $(-\infty, \infty)$ R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Part IV. ***Sketch a graph for each of the reciprocal functions on the interval [-2π, 2π].***

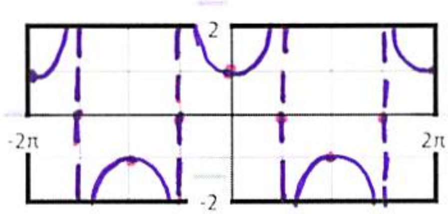
10. $y = \cot(x)$



X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	VA	0	VA	0	VA	0	VA	0	VA

D: All Real #'s Except π's R: $(-\infty, \infty)$

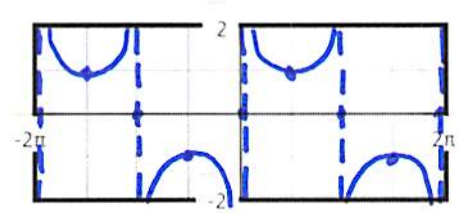
11. $y = \sec(x)$



X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	1	A	-1	A	1	A	-1	A	1

D: All Real #'s Except $\frac{\pi}{2}$'s R: $(-\infty, -1] \cup [1, \infty)$

12. $y = \csc(x)$



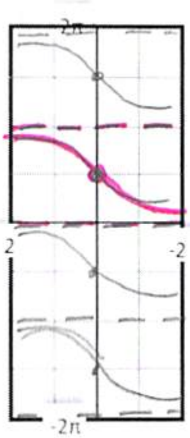
X	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	A	1	A	-1	A	1	A	-1	A

D: All Real #'s Except π's R: $(-\infty, -1] \cup [1, \infty)$

Using the guidelines in Part III. Sketch the arc-function for each of the reciprocal functions.

13. $y = arccot(x)$

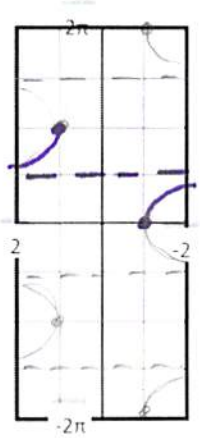
X	Y
HA	-2π
0	$-\frac{3\pi}{2}$
HA	$-\pi$
0	$-\frac{\pi}{2}$
HA	0
0	$\frac{\pi}{2}$
HA	π
0	$\frac{3\pi}{2}$
HA	2π



D: $(-\infty, \infty)$ R: $(0, \pi)$

14. $y = arcsec(x)$

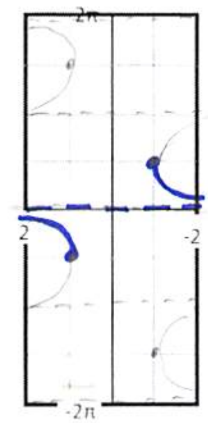
X	Y
1	-2π
HA	$-\frac{3\pi}{2}$
-1	$-\pi$
HA	$-\frac{\pi}{2}$
1	0
HA	$\frac{\pi}{2}$
-1	π
HA	$\frac{3\pi}{2}$
1	2π



D: $(-\infty, -1] \cup [1, \infty)$ R: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

15. $y = arccsc(x)$

X	Y
HA	-2π
1	$-\frac{3\pi}{2}$
HA	$-\pi$
-1	$-\frac{\pi}{2}$
HA	0
1	$\frac{\pi}{2}$
HA	π
-1	$\frac{3\pi}{2}$
HA	2π



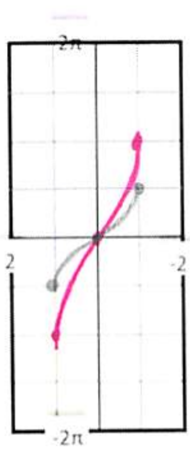
D: $(-\infty, -1] \cup [-1, \infty)$ R: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

Part V. Transformations. Given the function $\boldsymbol{y = a \cdot arcfunction(b(x-h) + k}$, recall that a and k indicate vertical changes and b and h indicate horizontal changes to the graph of the parent function.

NOTE: *Arcfunctions are not periodic (do not repeat). Thus, b does not change an arcfunction in the same way it does the original trigonometric function. It simply represents a horizontal stretch or compression.*

Use what you know about transformations of all functions to provide an accurate sketch of each of the following. Indicate the domain and range of each function. *You may make tables if you wish, but it is not required.*

16. $y = 2\arcsin(x)$

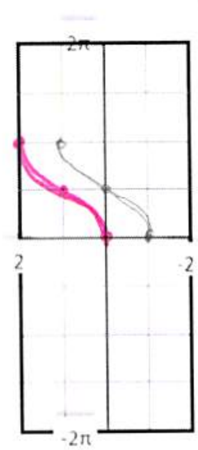


y=arcsin x

D: $[-1, 1]$ R: $[-\pi, \pi]$

17. $y = arc\cos(x+1)$

Left 1

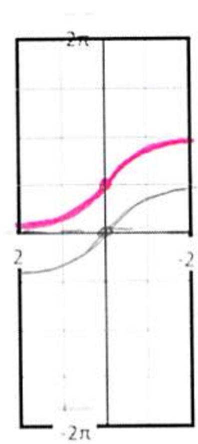


y=arccos x

D: $[-2, 0]$ R: $[0, \pi]$

18. $y = arc\tan(x) + \frac{\pi}{2}$

up $\frac{\pi}{2}$



D: $(-\infty, \infty)$ R: $(0, \pi)$