**Polar Graphs Exploration** Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Group 1: Circles**

In addition to circles of the form $r=a$, the equation of a **circle** can also be of the form $r=\pm a\sin(θ)$ or $r=\pm a\cos(θ)$ .

For each polar equation of a circle shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the circle.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=6\cos(θ)$$ | $$θ$$ | $$r=6\cos(θ)$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=-4\sin(θ)$$ | $$θ$$ | $$r=-4\sin(θ)$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |

Graph each of the above equations on your graphing calculator in polar mode. Verify that you have sketched your graphs correctly.

Given equations of the form $r=\pm a\sin(θ)$ or $r=\pm a\cos(θ)$, how do each of the following values affect the appearance of the graph of the circle?

* the value of $a$
* the sign of $a$
* $\sin(θ)$ versus $\cos(θ)$

**Group 2: Cardioids**

**Cardioids** are characterized by equations of the form $r=a\left(1\pm \cos(θ)\right)$ or $r=a\left(1\pm \sin(θ)\right)$ , where $a>0$. The graph of a cardioid always passes through the pole.

For the polar equation of the cardioid shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the cardioid.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=3\left(1-\sin(θ)\right)$$ | $$θ$$ | $$r=3\left(1-\sin(θ)\right)$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |

Graph the above equation on your graphing calculator in polar mode. Verify that you have sketched your graph correctly.

How would you describe the general shape of a cardioid?

On your calculator, experiment by graphing different cardioids in order to answer the question below.

Given equations of the form $r=a\left(1\pm \cos(θ)\right)$ or $r=a\left(1\pm \sin(θ)\right)$, how do each of the following values affect the appearance of the graph of the cardioid?

* the value of $a$
* the sign in the middle
* $\sin(θ)$ versus $\cos(θ)$

**Group 3: Limaçons**

**Limaçons** are characterized by equations of the form $r=a\pm b\cos(θ)$ or $r=a\pm b\sin(θ)$ , where $a,b>0$ and $a\ne b$.

For each polar equation of a limaçon shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the limaçon.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=3+2\cos(θ)$$ | $$θ$$ | $$r=3+2\cos(θ)$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=2-4\sin(θ)$$ | $$θ$$ | $$r=2-4\sin(θ)$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |

Graph each of the above equations on your graphing calculator in polar mode. Verify that you have sketched your graphs correctly.

How would you describe the general shape of a limaçon?

On your calculator, experiment by graphing different limaçons in order to answer the question below.

Given equations of the form $r=a\pm b\cos(θ)$ or $r=a\pm b\sin(θ)$, how do each of the following values affect the appearance of the graph of the limaçon?

* $a>b$ versus $a<b$
* the sign in the middle
* $\sin(θ)$ versus $\cos(θ)$

**Group 4: Roses**

**Rose** curves are characterized by equations of the form $r=a\cos((nθ))$ or $r=a\sin((nθ))$ , where $a\ne 0$.

For the polar equation of the rose curve shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the rose curve.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=5\cos((2θ))$$ | $$θ$$ | $$r=5\cos((2θ))$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |

Graph the above equation on your graphing calculator in polar mode. Verify that you have sketched your graph correctly.

How would you describe the general shape of a rose curve?

On your calculator, experiment by graphing different rose curves in order to answer the question below.

Given equations of the form $r=a\cos((nθ))$ or $r=a\sin((nθ))$, how do each of the following values affect the appearance of the graph of the rose curve?

* the value of $a$
* the value of $n$ (make special note of when $n$ is even vs. odd)
* $\sin(θ)$ versus $\cos(θ)$

**Group 5: Lemniscate**

**Lemniscates** are characterized by equations of the form $r^{2}=a^{2}\cos((2θ))$ or $r^{2}=a^{2}\sin((2θ))$ , where $a\ne 0$.

For the polar equation of the lemniscate shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the lemniscate.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r^{2}=25\sin((2θ))$$ | $$θ$$ | $$r^{2}=25\sin((2θ))$$ |
| $$0$$ |  | $$π$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{7π}/\_{6}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{11π}/\_{6}$$ |  |

Graph the above equation on your graphing calculator in polar mode. Verify that you have sketched your graph correctly.

How would you describe the general shape of a lemniscate?

On your calculator, experiment by graphing different lemniscate in order to answer the question below.

Given equations of the form $r^{2}=a^{2}\cos((2θ))$ or $r^{2}=a^{2}\sin((2θ))$, how do each of the following values affect the appearance of the graph of the lemniscate?

* the value of $a$
* $\sin(θ)$ versus $\cos(θ)$

**Group 6: Logarithmic Spirals**

**Logarithmic spirals** are characterized by equations of the form $r=b^{aθ}$ , where $a,b\ne 0$.

For the polar equation of the logarithmic spiral shown below, calculate the value of $r$ (rounded to the hundredths) for each standard angle shown. Then plot the points as accurately as possible in order to sketch the graph of the spiral.



|  |  |  |  |
| --- | --- | --- | --- |
| $$θ$$ | $$r=e^{^{θ}/\_{5}}$$ | $$θ$$ | $$r=e^{^{θ}/\_{5}}$$ |
| $$0$$ |  | $$^{5π}/\_{4}$$ |  |
| $$^{π}/\_{6}$$ |  | $$^{4π}/\_{3}$$ |  |
| $$^{π}/\_{4}$$ |  | $$^{3π}/\_{2}$$ |  |
| $$^{π}/\_{3}$$ |  | $$^{5π}/\_{3}$$ |  |
| $$^{π}/\_{2}$$ |  | $$^{7π}/\_{4}$$ |  |
| $$^{2π}/\_{3}$$ |  | $$^{11π}/\_{6}$$ |  |
| $$^{3π}/\_{4}$$ |  | $$2π$$ |  |
| $$^{5π}/\_{6}$$ |  | $$^{13π}/\_{6}$$ |  |
| $$π$$ |  | $$^{9π}/\_{4}$$ |  |
| $$^{7π}/\_{6}$$ |  | $$^{7π}/\_{3}$$ |  |
|  |  | $$^{5π}/\_{2}$$ |  |

Graph the above equation on your graphing calculator in polar mode. Verify that you have sketched your graph correctly.

How would you describe the general shape of a logarithmic spiral?

Why might this type of polar equation be called a logarithmic spiral? How does the appearance of this spiral differ from some other spirals you have seen in the past?

On your calculator, experiment by graphing different logarithmic spirals in order to answer the question below.

Given equations of the form $=b^{aθ}$ , how do each of the following values affect the appearance of the graph of the logarithmic spiral?

* the value of $b$
* the value of $a$