

Name _____ Date _____

Day 7 – Dilations and Combinations

Dilations are a resizing of the image. They change the lengths of the segments but NOT the angles. Unlike the other transformations we have learned about, dilation is not an isometry (a transformation in which the original figure and its image are congruent).

The first step to performing a dilation is to multiply by a scale factor. What is a scale factor? A **scale factor** is the number used to multiply the lengths of a figure to stretch or shrink it to a similar image.

- If a scale factor is less than 1, the resulting image will be a reduction.
- If a scale factor is greater than 1, the resulting image will be an enlargement.
- If a scale factor is equal to 1, the resulting image will be congruent.

TIP:

It may be helpful to convert fractions and percents to decimals to determine if the scale factor is greater than, less than, or equal to 1.

Practice: Determine if the following scale factors will result in an enlargement, reduction, or congruence:

- 1) $\frac{1}{4}$ 2) .75 3) 125% = 1.25 4) $\frac{15}{7} \approx 2.14$ 5) 100% = 1
- reduction reduction enlargement enlargement congruence

Now that we have developed an understanding of scale factors, we can begin performing dilations.

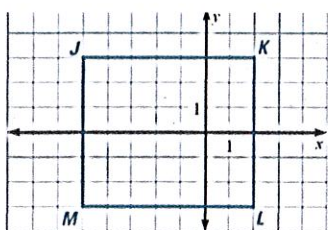
Steps for performing dilations:

- 1) Multiply BOTH coordinates by the given scale factor.
- 2) Simplify.
- 3) graph (if required).

Example:

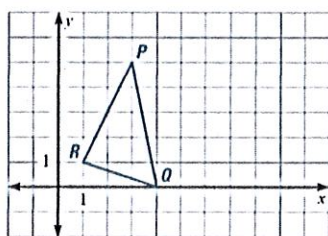
Use the given scale factor to find the coordinates of the vertices of the image of the polygon.

• $k = \frac{1}{2}$



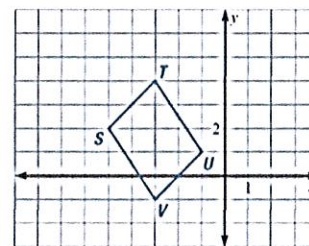
- J (-6, 3) → (-3, 1.5)
 K (2, 3) → (1, 1.5)
 L (2, -3) → (1, -1.5)
 M (-6, -3) → (-3, -1.5)

2) $k = 2$



- P (3, 5) → (6, 10)
 Q (4, 0) → (8, 0)
 R (1, 1) → (2, 2)

3) $k = 4$



- S (-5, 2) → (-20, 8)
 T (-3, 4) → (-12, 16)
 U (-1, 1) → (-4, 4)
 V (-3, -1) → (-12, -4)

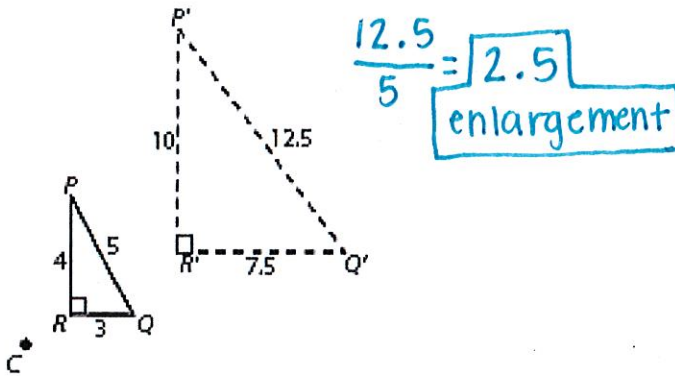
Finding the Scale Factor and Center of Dilation

Sometimes, we may be asked to work backwards. We may be given an image and pre image and be asked to find the scale factor. How can we do this?

The scale factor is the ratio of

$$\frac{\text{image distance}}{\text{pre image distance}} \text{ OR } \frac{\text{new image}}{\text{original image}}$$

Example: Determine the scale factor and whether the dilation is an enlargement, reduction, or congruency transformation. The dotted figure is the new image.

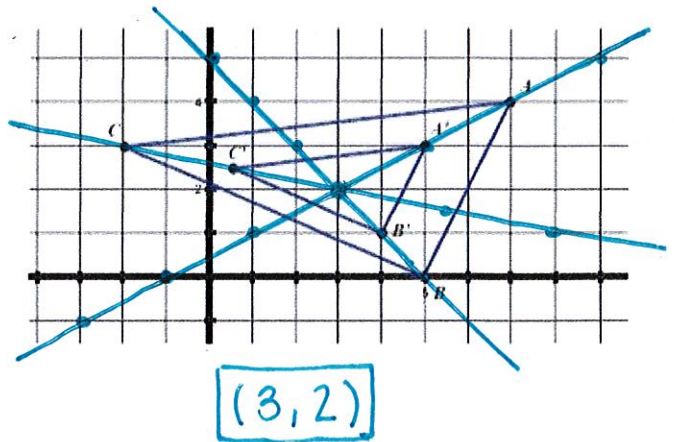


The **center of dilation** is a constant point on a surface from which all other points are either enlarged or compressed.

To find the center of dilation given two images (a pre image and image) we connect corresponding points from an image and pre image. **The intersection of the lines is the center of dilation.**

To ensure accuracy, use the slope between corresponding points to construct the lines.

Example: Find the center of dilation.



Compositions and Glide Reflections

When two or more transformations are combined to produce a single transformation, we call it a composition. The composition of 2 (or more) isometries is an isometry.

A **glide reflection** is the combination of a translation with a reflection. When performing a glide reflection, we perform each transformation in the respective order.

Examples:

- 1) Perform the glide reflection on $A(-3, 5)$.

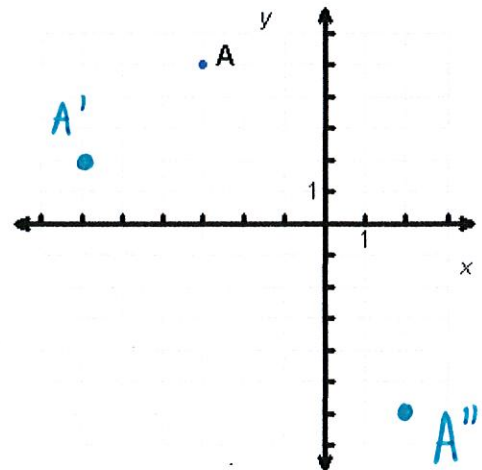
Translation: $(x, y) \rightarrow (x - 3, y - 3)$

$A'(-6, 2)$

Reflection: over $y = x$

$A''(2, -6)$

$A''(2, -6)$



2) Perform the following composition on \overline{CD} . $C(2,0)$, $D(3,3)$

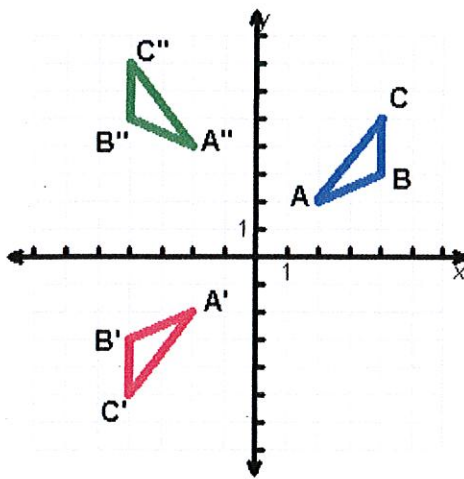
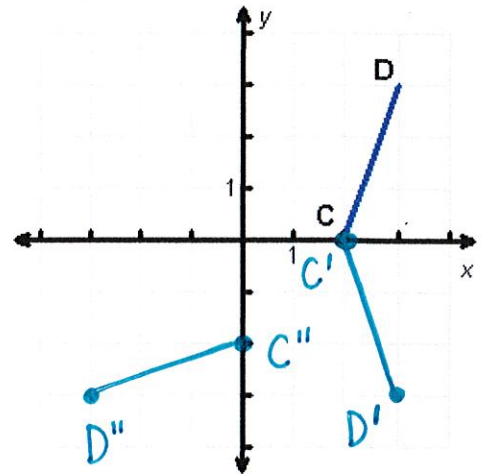
Reflection: over the x-axis

$C'(2,0)$ $D'(3,-3)$

Rotation: 270° counterclockwise about the origin

$C''(0,-2)$ $D''(-3,-3)$

$C''(0,-2)$
 $D''(-3,-3)$



3) Describe the composition.

- 1) rotation of 180° about the origin
- 2) reflect across $y=1$

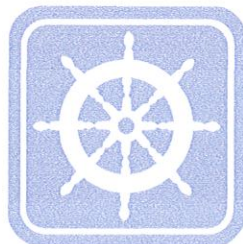
Rotational Symmetry

A figure has **rotational symmetry** if there is a center point around which the object is turned a certain number of degrees and the object looks the same. The degree of rotational symmetry that an object has is called its **order**. The order of rotational symmetry that an object has is the number of times that it fits onto itself during a full rotation of 360 degrees. To determine the **angle of rotation**, divide 360 degrees by its order.

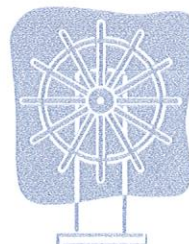
Example: Determine the order of the following.



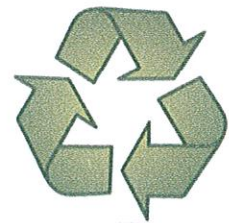
order: 6
AOR: $360/6 = 60^\circ$



order: 4
AOR: $360/4 = 90^\circ$



order: 1
AOR: $360/1 = 360^\circ$



order: 3
AOR: $360/3 = 120^\circ$