Name _____ Date _____

Day 5 – Midpoint and Partitioning a Line Segment

The **Midpoint Formula** allows you to find the **midpoint** or **center** between two points.

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Practice:

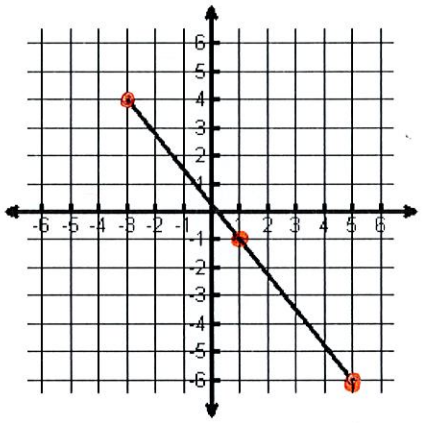
1. Find the midpoint between (1, -2) and (-3, 6).

$\left(\frac{1+(-3)}{2}, \frac{-2+6}{2}\right)$

$\left(\frac{-2}{2}, \frac{4}{2}\right)$

$(-1, 2)$

2. Find the midpoint between (-3, 4) and (5, -6).



$\left(\frac{-3+5}{2}, \frac{4+(-6)}{2}\right)$

$\left(\frac{2}{2}, \frac{-2}{2}\right)$

$(1, -1)$

3. M is the midpoint of segment AB. The coordinates of A are (-2, 3) and the coordinates of M are (1, 0). Find the coordinates of B.

x: $\frac{-2+x_2}{2} = 1$, $2 \cdot \frac{-2+x_2}{2} = 1 \cdot 2$, $-2+x_2 = 2$, $x_2 = 4$

y: $\frac{3+y_2}{2} = 0$, $2 \cdot \frac{3+y_2}{2} = 0 \cdot 2$, $3+y_2 = 0$, $y_2 = -3$

The coordinates of B are (4, -3)

4. B is the midpoint of segment AC. The coordinates of A are (-10, 4) and the coordinates of B are (-2,4). Find the coordinates of C.

x: $\frac{-10+x_2}{2} = -2$, $2 \cdot \frac{-10+x_2}{2} = -2 \cdot 2$, $-10+x_2 = -4$, $x_2 = 6$

y: $\frac{4+y_2}{2} = 4$, $2 \cdot \frac{4+y_2}{2} = 4 \cdot 2$, $4+y_2 = 8$, $y_2 = 4$

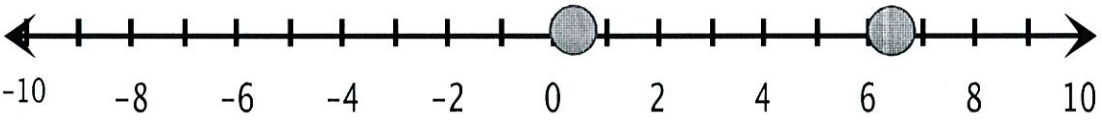
The coordinates of C are (6, 4)

Partitioning a Line Segment

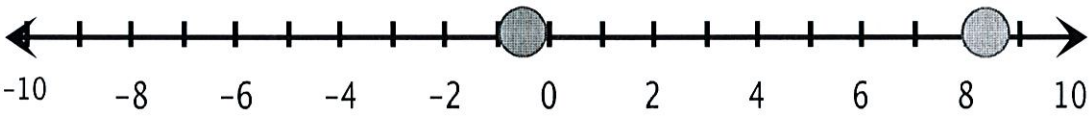
You just learned how to calculate the midpoint of a line segment. Think …

- How many segments does the midpoint split a segment into?
- Are these segments equal in length?

This is called a 1 to 1 ratio (1:1), which means the length of the first segment is one times as big (or equal to) as the second segment. Since the ratio is 1:1, you can also think of it as dividing a segment into two equal parts.



A 2:1 ratio would be interpreted as a segment being divided into three equal parts (2 + 1) with two equal parts representing the "2" in the ratio and the other remaining equal part representing the "1".



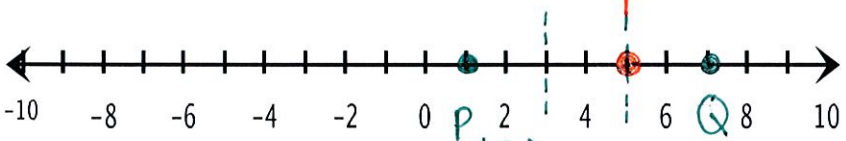
When we divide or separate a line segment, we are **partitioning the segment**. Today, we are going to learn how to partition a segment using a given ratio, other than 1:1 (midpoint). When partitioning a segment, it Is necessary to determine the total number of parts that the line segment must be divided into. In the following ratios below, determining the total number of parts:

a. 2:5 b. 3:5 c. 1:2 d. 3:8

7 8 3 11

Partitioning a Line Segment on a Number Line

1. Given: P is at 1 and Q is at 7. Find the point T, so that T partitions P to Q in a 2:1 ratio. a b



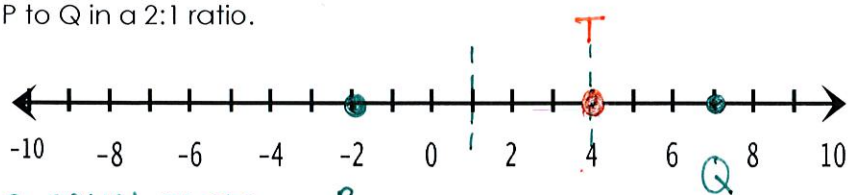
3 equal parts $T=(7-1)\left(\frac{2}{2+1}\right)+1$

$T = 6\left(\frac{2}{3}\right)+1$

$T = 4+1$

T is at 5

2. Given: P is at -2 and Q is at 7. Find the point T, so that T partitions P to Q in a 2:1 ratio.



3 equal parts

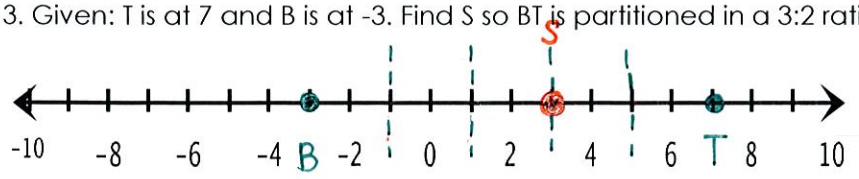
T is at 4

Partitioning in One Dimension:

1. Plot endpoints (x_1 & x_2)
2. Determine number of equal parts.
3. Determine length of segment.
4. Count the number of equal parts in the 1st part of the ratio and plot the point.

Formula: $(x_2 - x_1)\left(\frac{a}{a+b}\right) + x_1$

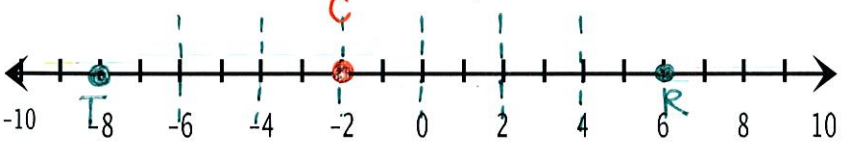
3. Given: T is at 7 and B is at -3. Find S so BT is partitioned in a 3:2 ratio.



5 equal parts

S is at 3

4. Given: T is at -8 and R is at 6. Find C, so that TC is three-sevenths of TR Ratio 3:4



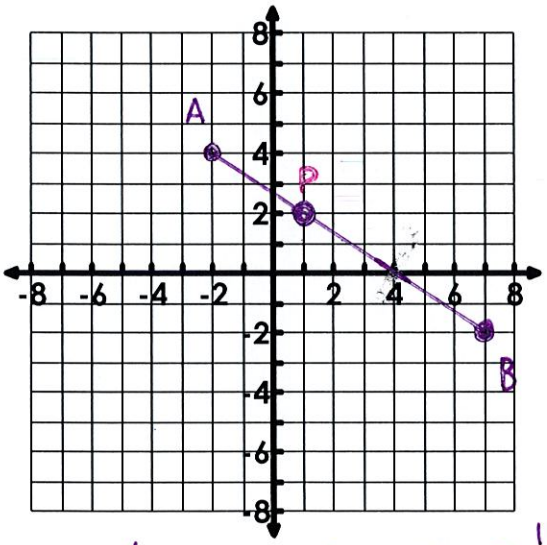
7 equal parts

C is at -2

Partitioning a Segment in Two Dimensions

Partitioning a segment in two dimensions means you are partitioning a line segment in a coordinate plane. It is very similar to partitioning a segment in one dimension except instead of multiplying your fraction by the length; you will be multiplying by the rise (y-coordinate) and run (x-coordinate) of the segment.

Example 1: Given the points A(-2, 4) and B(7, -2), find the coordinates of the point P on the line segment AB that partitions AB in the ratio 1:2.



Slope = $\frac{-6}{9}$ ratio fraction = $\frac{1}{3}$

$\frac{-6\,(1/3)}{9\,(1/3)} = \frac{-2}{3}$ ← new slope

(1, 2)

Partitioning in Two Dimensions:

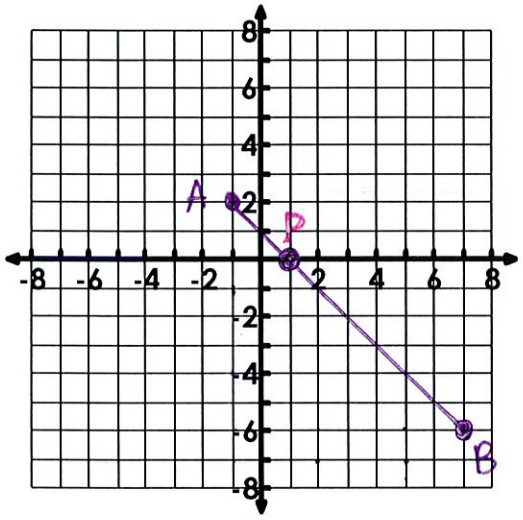
1. Plot points (x_1, y_1) & (x_2, y_2).
2. Determine your slope $(\frac{rise}{run})$.
3. Multiply the **rise** by the fraction that represents the first part of the ratio $(\frac{a}{a+b})$.
4. Multiply the **run** by the fraction that represents the first part of the ratio $(\frac{a}{a+b})$.
5. Go back to point A and plot a point using your new rise over run value.
6. The plotted point represents the given ratio.

Formula:

$$\left((x_2 - x_1)\left(\tfrac{a}{a+b}\right) + x_1,\ (y_2 - y_1)\left(\tfrac{a}{a+b}\right) + y_1\right)$$

* you may use the formula if you find it to be easier

Example 2: Given the points A(-1, 2) and B (7, -6), find the coordinates of the point P on the line segment AB that partitions AB in the ratio 1:3.

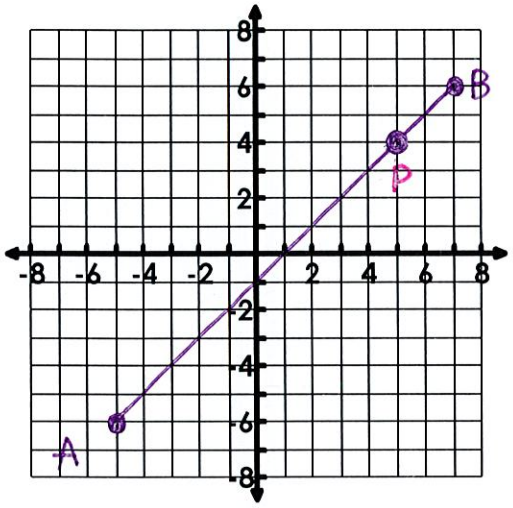


slope $= \frac{-8}{8}$ ratio fraction $= \frac{1}{4}$

$\frac{-8}{8} \frac{(1/4)}{(1/4)} = \frac{-2}{2}$

(1,0)

Example 3: Given the points A(-5, -6) and B (7, 6), find the coordinates of the point P on the line segment AB that partitions AB in the ratio 5:1.

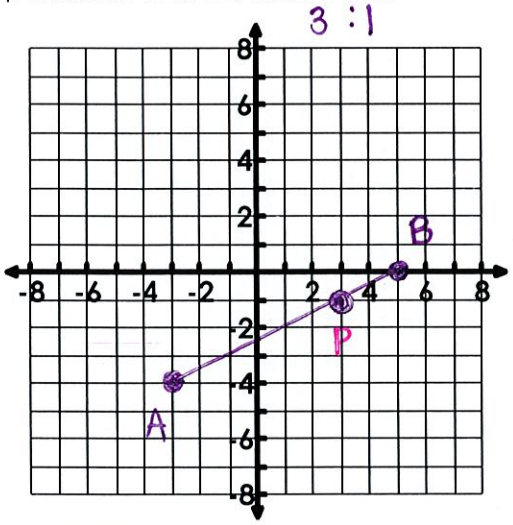


slope $= \frac{12}{12}$ ratio fraction $= \frac{5}{6}$

$\frac{12}{12} \frac{(5/6)}{(5/6)} = \frac{10}{10}$

(5,4)

Example 4: Given the points A(-3, -4) and B(5, 0), find the coordinates of the point P on the line segment AB that partitions AB in the ratio 3 to 1.



slope $= \frac{4}{8}$ ratio fraction $= \frac{3}{4}$

$\frac{4}{8} \frac{(3/4)}{(3/4)} = \frac{3}{6}$

(3,-1)