

Day 8 – Changing Forms of Circles

Like many other equations we have studied in the past (linear and quadratic), there are several different forms that an equation can be written. Circles are no exception. We have two forms to represent equations of circles: Standard Form and General Form.

The **standard form** is the most beneficial to us as it tells us the center and radius of a circle. You can think of standard form as “center-radius” form. The general form does not really serve any purpose other than for us to see that both x and y are squared and we have an equation of a circle.

The Standard Form of a Circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

General Form:

$$Ax^2 + By^2 + Cx + Dy + E = 0.$$

Converting from Standard Form to General Form

1. Multiply any binomials.
2. Combine like terms
3. Arrange terms so they are in the order of x^2 , y^2 , x , y , constant
4. Add or subtract the constant on the right side so the entire equation is set equal to 0

a. $x^2 + (y - 1)^2 = 4$

$$x^2 + (y - 1)(y - 1) = 4$$

$$x^2 + y^2 - 1y - 1y + 1 = 4$$

$$x^2 + y^2 - 2y + 1 = 4$$

$$x^2 + y^2 - 2y - 3 = 0$$

Try on your own:

c. $(x - 4)^2 + (y + 3)^2 = 36$

$$(x - 4)(x - 4) + (y + 3)(y + 3) = 36$$

$$x^2 - 4x - 4x + 16 + y^2 + 3y + 3y + 9 = 36$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 36$$

$$x^2 + y^2 - 8x + 6y + 25 = 36$$

$$x^2 + y^2 - 8x + 6y - 11 = 0$$

b. $(x - 5)^2 + (y + 2)^2 = 10$

$$(x - 5)(x - 5) + (y + 2)(y + 2) = 10$$

$$x^2 - 5x - 5x + 25 + y^2 + 2y + 2y + 4 = 10$$

$$x^2 - 10x + 25 + y^2 + 4y + 4 = 10$$

$$x^2 + y^2 - 10x + 4y + 29 = 10$$

$$x^2 + y^2 - 10x + 4y + 19 = 0$$

Rewriting Equations of Circles in Standard Form

As you just practiced, to go from standard form to general form requires you to multiply the binomials and combine like terms. If an equation of a circle is given to you in general form, your first instinct should be to convert it to standard form so you can identify the radius and center.

To convert from general to standard form for, you must do the following to the equation given below:

Steps	Example
1. Factor out the greatest common factor (GCF) and then divide both sides by the GCF. (This will cause the GCF to "disappear" on the left side and allow the right side to remain equal to zero.)	$x^2 + y^2 - 2x - 6y + 6 = 0$ $x^2 - 2x + \underline{1} + y^2 - 6y + \underline{9} = -6 + \underline{1} + \underline{9}$
2. Group your "x" terms and "y" terms together and place a + _____ after the x and y terms. (You will be completing the square eventually)	$(\frac{b}{2})^2 = (\frac{-2}{2})^2 = (-1)^2 = 1$ $(\frac{b}{2})^2 = (\frac{-6}{2})^2 = (-3)^2 = 9$
3. Add or subtract any constants (plain numbers) to the other side.	$x^2 - 2x + 1 + y^2 - 6y + 9 = 4$
4. Use the method of completing the square for both the x and y terms. Whichever numbers you added in both your blanks must also be added to the right side. (Remember, when you are completing the square, you are trying to create perfect square trinomials that you can eventually factor.)	$(x - 1)^2 + (y - 3)^2 = 4$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $(x - 1)^2 + (y - 3)^2 = 4$ </div>
5. Factor both perfect square trinomials and write in factor form. Add the constants on the right.	

Rewrite each equation in standard form. Then identify the center and radius.

a. $x^2 + y^2 - 8x + 7 = 0$

$x^2 - 8x + \underline{16} + y^2 + \underline{0}y + \underline{0} = -7 + \underline{16} + \underline{0}$

$(\frac{b}{2})^2 = (\frac{-8}{2})^2 = 16$
 $(\frac{b}{2})^2 = (\frac{0}{2})^2 = 0$

$(x - 4)^2 + (y + 0)^2 = 9$

$(x - 4)^2 + y^2 = 9$

Center: (4, 0)

Radius: 3

b. $x^2 + y^2 + 4x - 6y - 3 = 0$

$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = 3 + \underline{4} + \underline{9}$

$(\frac{b}{2})^2 = (\frac{4}{2})^2 = 4$
 $(\frac{b}{2})^2 = (\frac{-6}{2})^2 = 9$

$(x + 2)^2 + (y - 3)^2 = 16$

$(x + 2)^2 + (y - 3)^2 = 16$

Center: (-2, 3)

Radius: 4

c. $2x^2 + 2y^2 - 16x + 4y + 20 = 0$

$x^2 + y^2 - 8x + 2y + 10 = 0$

$x^2 - 8x + \underline{16} + y^2 + 2y + \underline{1} = -10 + \underline{16} + \underline{1}$

$(\frac{b}{2})^2 = (\frac{-8}{2})^2 = 16$
 $(\frac{b}{2})^2 = (\frac{2}{2})^2 = 1$

$(x - 4)^2 + (y + 1)^2 = 7$

$(x - 4)^2 + (y + 1)^2 = 7$

Center: (4, -1)

Radius: $\sqrt{7} \approx 2.6$